Low-energy features of SU(2) Yang-Mills theory with light gluinos

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We report on the latest results of the low-lying spectrum of bound states in SU(2) Yang-Mills theory with light gluinos. The behavior of the disconnected contributions in the critical region is analyzed. A first investigation of a three-gluino state is also discussed.

1. Introduction

The numerical simulation we report on aims at a better understanding of the non-perturbative low-energy features of supersymmetric gauge theories. We concentrate on the simplest supersymmetric gauge theory, namely SU(2),$N = 1$ super-Yang-Mills. This model contains, in addition to the gauge field a massless Majorana fermion in the adjoint representation (called gluino). For the theoretical motivation of this investigation see [1–3] and references therein.

2. Lattice formulation

We regularize the theory by the Wilson action as proposed in [4]. Supersymmetry is broken, both by the lattice regularization and the introduction of a mass term for the gluino. The action contains two bare parameters: the gauge coupling β and the hopping parameter K (bare gluino mass). Supersymmetry is expected to be restored by tuning the bare parameters to their critical values [4]. The path-integral for Majorana fermions is a Pfaffian

$$\int [d\psi] e^{-\frac{1}{2}\psi_a (CQ)_{ab}\psi_b} = \mathrm{Pf(CQ)}, \tag{1}$$

where Q is the Wilson fermion matrix in the adjoint representation (see for example [3]), and C the charge conjugation matrix. The Pfaffian satisfies

$$\mathrm{Pf(CQ)}^2 = \det(\mathrm{CQ}) = \det \mathrm{Q} = \det(\tilde{\mathrm{Q}}). \tag{2}$$

$\tilde{Q}$ is the hermitean fermion matrix $\tilde{Q} = \gamma_5 Q$ with doubly degenerate real eigenvalues, ($\det(Q) \geq 0$). In practice we have simulated with weight $\det(Q)^{\frac{1}{2}}$. This may lead to a sign problem. However, in [3] it is found that sign flips are rare.

3. The low-lying spectrum

A basic assumption about the low-energy dynamics of super-Yang-Mills theory is confinement, as supported by the non-vanishing string tension [3]. Therefore the low-lying spectrum consists of color singlets as in QCD. In the SUSY-limit of zero gluino mass the states should be organized in degenerate multiplets. In analogy to QCD we consider scalar and pseudoscalar mesons and glueballs. To complete the supermultiplet a spin $\frac{1}{2}$ gluino-glue particle is also considered. In detail these particles and some of the corresponding operators are:

- Scalar meson (a-f0): $\phi_s = \bar{\psi}\psi$,
- Pseudoscalar meson (a-η'): $\phi_p = \bar{\psi}\gamma_5\psi$,

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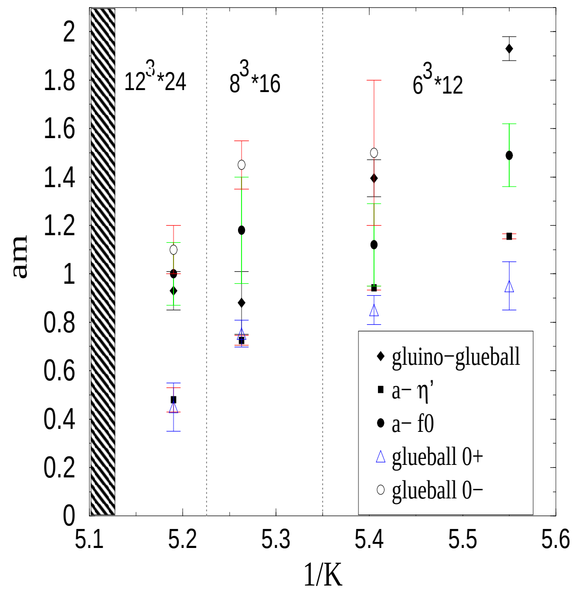

Figure 1. The lightest bound state masses in lattice units as function of the bare gluino mass parameter $1/K$. The shaded area at $K = 0.1955(5)$ is where zero gluino mass and supersymmetry are expected [5].

- Gluino-glue state $:\chi_\alpha = \sum_{kl} Tr(\tau_r U_{kl})\psi^r_\alpha$,
- 0^+ glueball,
- 0^- glueball.

For the gluino-glue state and the glueball masses blocking and smearing was used. The results are displayed in fig.1. The presumable existence of a second multiplet requires yet another spin $\frac{1}{2}$ particle. The search for this state is an open issue.

4. A look at the $a-\eta'$ in the critical region

The correlator of the $a-\eta'$ consists of a disconnected and a connected part,

$$C(t) = -2C(t)_{\mathrm{conn}} + C(t)_{\mathrm{disconn}}.$$

In QCD, $C(t)_{\mathrm{conn}}$ gives rise to the π-mass and $C(t)$ to the η'-mass, so that

$$R(t) = C(t)/C(t)_{\mathrm{disconn}}$$

is expected to decrease as we approach the chiral limit. In order to investigate whether this is also

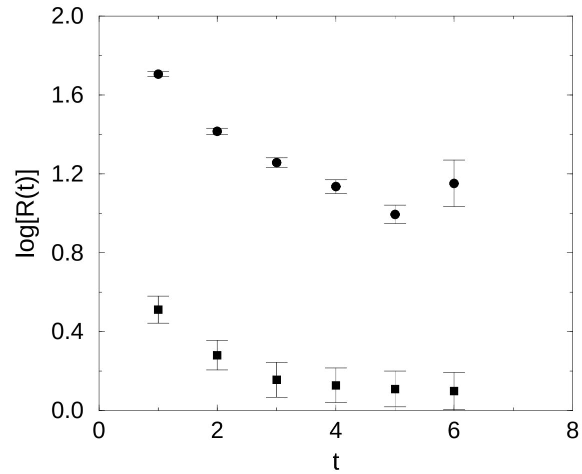

Figure 2. R(t) as defined in the text at K=0.1925 (circles) and K=0.196(squares)

true in our case, we plot $R(t)$ in fig.2. For $K = 0.1925$ and $K = 0.196$ we observe that indeed $R(t)$ demonstrates a QCD-like behavior.

5. Investigation of a three-gluino state

Three-gluino states[3] can also be constructed in analogy to QCD baryons. This holds also for SU(2) since the fermions are in the adjoint representation. In this case a possible choice for the wave function is

$$\phi^\alpha(x) = \epsilon_{abc}(C\gamma_4)_{\beta\gamma}\psi(x)^\alpha_a\psi(x)^\beta_b\psi(x)^\gamma_c. \qquad (3)$$

This wave function which is antisymmetric in color and symmetric in spin, carries spin $\frac{3}{2}$.
For SU(3) color additional possibilities are obtained by a symmetric color coupling

$$\phi'^\alpha(x) = d_{abc}(C\gamma_5)_{\beta\gamma}\psi(x)^\alpha_a\psi(x)^\beta_b\psi(x)^\gamma_c,$$
$$\phi''^\alpha(x) = d_{abc}(C)_{\beta\gamma}\psi(x)^\alpha_a\psi(x)^\beta_b\psi(x)^\gamma_c.$$

The propagator of such a state has basically two contributions displayed in fig.3. The correlation function $<\bar{\phi^\alpha}\phi^\alpha>$ for the wave function eq.(3) has the following form:

$$C(x,y) = -\epsilon_{a'b'c'}\epsilon_{abc}(C\gamma_4)_{\beta'\gamma'}(C\gamma_4)_{\beta\gamma}*$$
$$\{\, 2\Delta^{ya'\alpha'}_{xa\alpha}\Delta^{yb'\beta'}_{xb\beta}\Delta^{yc'\gamma'}_{xc\gamma} + 4\Delta^{yb'\beta'}_{xa\alpha}\Delta^{yc'\gamma'}_{xb\beta}\Delta^{ya'\alpha'}_{xc\gamma}$$

[3] We would like to thank A.González-Arroyo for a clarifying discussion on the spin content of these particles.

three-gluino propagator

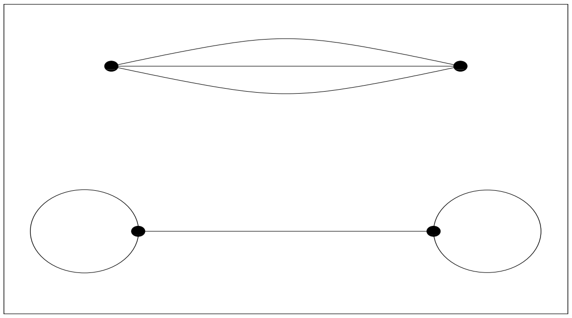

Figure 3. Contributions to the propagator of a three-gluino state. The second contribution arises, since contractions of the form $\psi(x)\psi(x)$ are allowed for Majorana fermions.

$$+2\Delta^{xb\beta}_{xa\alpha}\Delta^{ya'\alpha'}_{xc\delta}\Delta^{yb'\beta'}_{yc'\delta'}C_{\gamma\delta}C_{\delta'\gamma'} + 4\Delta^{xb\beta}_{xa\alpha}\Delta^{xc\gamma}_{yb'\beta'}\Delta^{ya'\alpha'}_{yc'\gamma'}$$
$$+\Delta^{ya'\alpha'}_{xa\alpha}\Delta^{xc\delta}_{xb\beta}\Delta^{yb'\beta'}_{yc'\delta'}C_{\gamma\delta}C_{\delta'\gamma'}$$
$$+2\Delta^{yc'\delta'}_{xa\alpha}\Delta^{xc\delta}_{xb\beta}\Delta^{ya'\alpha'}_{yb'\beta'}C_{\gamma\delta}C_{\delta'\gamma'}\ \} \,,$$

where $\Delta = Q^{-1}$ is the gluino propagator. The last four terms pertaining to the second "spectacles" graph can be evaluated by "gauge averaging" in analogy to the volume source method [6].

5.1. Evaluation of the spectacles graph

We now show how to evaluate the second graph of fig.3. With Ω_x the gauge transformation in the fundamental representation, we see that the gauge transformation in the adjoint, defined as $G_{x,ab}(\Omega) = [G^{-1}_{x,ab}]^T = 2Tr[\tau_a\Omega^{-1}(x)\tau_b\Omega(x)]$, obeys

$$\int d\Omega G_{a_1b_1} = 0,$$
$$\int d\Omega G_{a_1b_1}G_{a_2b_2}G_{a_3b_3} = \frac{1}{6}\epsilon_{a_1a_2a_3}\epsilon_{b_1b_2b_3}. \qquad (4)$$

The propagator Δ transforms under a gauge transformation as

$$\Delta^{yb}_{xa} \to G^{-1}_{x,aa'}\Delta^{yb'}_{xa'}G_{y,b'b}. \qquad (5)$$

These are the necessary ingredients for an evaluation of the second graph. We have to calculate for example (spinor indices are left out for simplicity)

$$\tilde{C}(x,y) \equiv \Delta^{ya'}_{yc'}\Delta^{xc}_{yb'}\Delta^{xb}_{xa}\epsilon_{abc}\epsilon_{a'b'c'}.$$

First we compute the vector

$$W_{zb',x} = \Delta^{xc}_{zb'}\Delta^{xb}_{xa}\epsilon_{abc},$$

for a fixed site x and all sites z. Next we observe that, with the help of eqs.(4) and (5), we find the identity

$$< \Delta^{za'}_{yc'}W_{zb',x} >= \frac{1}{6}\delta_{zy}\epsilon_{a'b'c'}\epsilon_{abc} < \Delta^{ya}_{yc}W_{yb,x} > .$$

Composing now the "shifted" vector $W^{\text{shifted}}_{xc,y}$,

$$W^{\text{shifted}}_{zb',x} = W_{zb'-1,x} - W_{zb'+1,x}$$

(with $W_{x4,y} = W_{x1,y}$, $W_{x0,y} = W_{x3,y}$) it can be shown that

$$\sum_{y,c',b'} < \Delta^{zb'}_{yc'}W^{\text{shifted}}_{zb',x} >=< \tilde{C}(x,y) > .$$

To evaluate the l.h.s. of this relation numerically only one additional inversion is needed with $W^{\text{shifted}}_{zb',x}$ as the source. In this way $\left\langle \tilde{C}(x,y) \right\rangle$ is obtained from a given x to all y by two inversions of the fermion matrix Q. An analysis of the mass of the particle characterized by eq.(3) is currently under way.

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